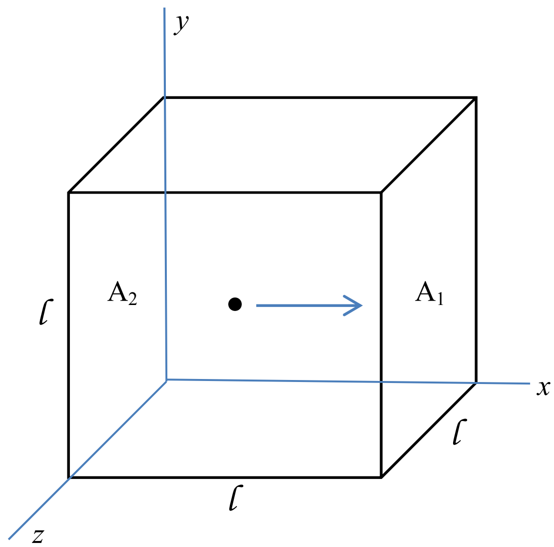
Kinetic Molecular Theory Equations:



Kinetic Calculation of Pressure (particle in box)

v = v_x, v_y, v_z Assume no velocity in the v_y or v_z direction

Momentum change
$\Delta p = p_f - p_i = (-mv_x) - (mv_x) = -2mv_x$

Momentum imparted to side $A_1 = 2mv_x$ since momentum is conserved

After bouncing off of A_1 the particle returns to A_2 (assume without running into other molecules)

Time required crossing the cube ℓ / v_x

Round trip time = $2\ell / v_x$ (neglecting negligible impact time)

Number of collisions per unit time with wall $A_1 = v_x / 2\ell$ (units of 1/time = frequency)

Rate at which momentum is transferred to A_1

$2mv_x(v_x/2\ell) = mv_x^2 / \ell$

Total force on A_1 due to all gas molecules (sum up mv_x^2/ℓ)

$F = (m / \ell)\,(v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \ldots)$

The pressure is the total force divided by the area (ℓ^2)

$P = (m / \ell^3)\,(v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \ldots)$

If N = the total number of particles in the container

$$P = \frac{mN}{V}\left(\frac{{v_{x1}}^2 + {v_{x2}}^2 + \cdots}{N}\right)$$

mN = total mass of the gas   mN/V = mass per unit volume (density, ρ)

$$\left(\frac{{v_{x1}}^2 + {v_{x2}}^2 + \cdots}{N}\right)$$

is the average value of v_x^2 defined now to be $\overline{{v_x}^2}$

$$P = \rho\overline{{v_x}^2}$$

For any particle $v^2 = v_x^2 + v_y^2 + v_z^2$.

Due to random motion, average values for v_x^2, v_y^2 and v_z^2 are the same and equal to 1/3 the average value of v^2. There is no preference along any of the axes.

Therefore

$$P = \rho\overline{{v_x}^2} = \frac{1}{3}\rho\overline{v^2}$$

$\sqrt{\overline{v^2}}$ = root-mean-square (RMS) speed of the molecules (similar to an "average" speed.)

Solving $v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3P}{\rho}}$

(Ignoring small fluctuations over small regions of space and short periods of time.)

Kinetic Interpretation of Temperature

Multiplying an earlier equation on both sides by V (volume)

$$PV = \frac{1}{3}\rho V {v_{rms}}^2$$

ρV = total mass of the gas.   Total mass is also written as nM   (n is moles; M is molar mass)

Substituting

$$PV = \frac{1}{3}nM{v_{rms}}^2$$

The right side of the equation is two-thirds the total kinetic energy of translation of the molecule

$$PV = \frac{2}{3}\left(\frac{1}{2}nM{v_{rms}}^2\right)$$

The ideal gas law expression is PV = nRT

Combining equations:

$$\frac{1}{2}M{v_{rms}}^2 = \frac{3}{2}RT$$

The total translational kinetic energy per mole of the molecules of an ideal gas is proportional to the temperature.

The temperature of a gas is related to the total translational kinetic energy measured with respect to the center of mass of the gas.

Dividing both sides by Avogadro's number N_A (for which M/N_A = m, is the mass of a single molecule)

$$\frac{1}{2}\left(\frac{M}{N_A}\right){v_{rms}}^2 = \frac{1}{2}m{v_{rms}}^2 = \frac{3}{2}\left(\frac{R}{N_A}\right)T$$

$\frac{1}{2}m{v_{rms}}^2$ is the average translational kinetic energy per molecule.
The ratio $\frac{R}{N_A}$ is given the symbol k and is called the Boltzmann constant

$$\frac{1}{2}m{v_{rms}}^2 = \frac{3}{2}kT$$

k = $1.381\text{x}10^{-23}$ J/molecule K

Conclusion: At the same temperature T, the ratio of the rms speeds of molecules of two different gases is equal to the square root of the inverse ratio of their masses.
i.e.

$$T = \frac{2}{3k}\frac{m_1{v_{1rms}}^2}{2} = \frac{2}{3k}\frac{m_2{v_{2rms}}^2}{2}$$

$$\frac{v_{1rms}}{v_{2rms}} = \sqrt{\frac{m_2}{m_1}}$$